PROBLEMS (variable acceleration, linear & angular motion & tangential & normal acceleration)

① The angle of rotation of a body is given by the equation, $\theta = 2t^3 - 5t^2 + 8t + 6$ where θ is in radians and t in seconds. Determine angular velocity and angular acceleration of the body, when $t = 0$ & $t = 4$ s.

Solution

$$\theta = 2t^3 - 5t^2 + 8t + 6$$

$$\omega = \frac{d\theta}{dt} = 6t^2 - 10t + 8$$

$$\alpha = \frac{d\omega}{dt} = 12t - 10$$

At $t = 0 \Rightarrow \omega = 6 \times 0 - 10 \times 0 + 8 = \underline{8 \text{ rad/s}}$

$\alpha = 12 \times 0 - 10 = \underline{-10 \text{ rad/s}^2}$

At $t = 4 \Rightarrow \omega = 6 \times (4)^2 - 10 \times 4 + 8 = \underline{64 \text{ rad/s}}$

$\alpha = 12 \times 4 - 10 = \underline{38 \text{ rad/s}^2}$

② The angle of rotation of a body is given as a function of time by the equation, $\theta = \theta_0 + at + bt^2$ where θ_0 is initial angular displacement and a & b are constants. Obtain the general expression for a) angular velocity; b) angular acceleration, if initial angular velocity be 3π rad/s and after 2 sec, the angular velocity is 8π rad/s. Determine the constants a & b.

Solution

$$\theta = \theta_0 + at + bt^2$$

$$\omega = \frac{d\theta}{dt} = a + 2bt$$

$$\alpha = \frac{d\omega}{dt} = 2b$$

At $t = 0$; $\omega = 3\pi$ rad/s.

$\therefore 3\pi = a + 0$

$\therefore a = 3\pi$

At $t = 2s$; $\omega = 8\pi$ rad/s.

$\therefore 8\pi = 3\pi + 2b \times 4$

$5\pi = 4b$

$\therefore b = \frac{5\pi}{4}$

$\therefore \theta = \theta_0 + 3\pi t + \frac{5}{4}\pi t^2$

$\omega = 3\pi + \frac{5}{2}\pi t$

$\alpha = \frac{5}{2}\pi$

③ The motion of a gear is defined by the relation $\theta = 4t^3 - 6t^2 + 9t + 20$, where θ is in radians and t is in s. Determine the angular displacement, angular velocity and angular acceleration at time, $t = 3$ sec

Solution

$\theta = 4t^3 - 6t^2 + 9t + 20$

$\omega = 12t^2 - 12t + 9$

$\alpha = 24t - 12$

At $t = 3$ sec; $\Rightarrow \theta = 4 \times (3)^3 - 6 \times (3)^2 + 9 \times 3 + 20$

$= 101$ radians

$\omega = 12 \times (3)^2 - 12 \times 3 + 9$

$= 81$ rad/s

$\alpha = 24 \times 3 - 12 = 60$ rad/s^2

④ The angular acceleration of a wheel is given by $\alpha = 12 - t$, where α is in rad/s^2 and t in seconds. If the angular velocity of the wheel is 60 rad/s at the end of 4 sec. Determine the angular velocity at the end of 6 sec. How many revolutions take place in this 6 sec.

Solution

$\alpha = 12 - t$

$\omega = \int (12 - t)\, dt = 12t - \frac{t^2}{2} + C_1$

$$\theta = \frac{12t^2}{2} - \frac{t^3}{6} + C_1 t + C_2$$

At, $t=0$; $\theta=0$

$\therefore 0 = 0 + C_2 \Rightarrow C_2 = 0$

At, $t=4$; $\omega = 60$ rad/s.

$$60 = 12\times4 - \frac{(4)^2}{2} + C_1$$

$\therefore C_1 = 20$

ii; $\omega = 12t - \frac{t^2}{2} + 20$

At $t = 6$ sec $\Rightarrow \omega = 12\times6 - \frac{6^2}{2} + 20$

$= \underline{\underline{74 \text{ rad/s}}}$

$$\theta = \frac{12t^2}{2} - \frac{t^3}{6} + 20t + 0$$

At $t=6$ sec; $\Rightarrow \theta = \frac{12\times6^2}{2} - \frac{6^3}{6} + 20\times6$

$= \underline{\underline{300 \text{ rad}}}$.

No. of revolutions $= \frac{300}{2\pi} = \underline{\underline{47.75}}$ rev.

(5) A motor has an angular acceleration which is directly proportional to the time. The initial angular velocity is zero. After 4 sec from start, the motor has completed 6 revolutions. Obtain the equation of motion of the motor and determine the angular velocity at the end of 2 sec.

Solution

$\alpha \propto t$

$\alpha = kt$

$$\omega = \frac{kt^2}{2} + C_1$$

$$\theta = \frac{kt^3}{6} + C_1 t + C_2.$$

At $t=0$; $\theta = 0$.

$0 = 0 + 0 + C_2 \Rightarrow C_2 = 0$

At $t=0$; $\omega=0$

ie; $0 = 0 + C_1 \Rightarrow C_1 = 0$

At $t = 4s$, $\theta = 6\ rev = 6 \times 2\pi$ rad.

ie; $6 \times 2\pi = \frac{k \times (4)^3}{6} + 0$

$\therefore k = 1.125\pi = \underline{3.53}$

$\therefore$ Equation of motion $\Rightarrow$ $\underline{\alpha = 3.53t}$

At $t = 2sec$; $\Rightarrow$ $\omega = \frac{3.53t^2}{2} = \frac{3.53 \times (2)^2}{2}$

$= \underline{\underline{7.06\ rad/s}}$

⑥ A passenger car is travelling at 65 km/hr on a level road. The distance from the road to the centre of the wheel is 30 cm. Determine;

a) the magnitude of angular velocity of the wheels

b) the magnitude of the constant angular deceleration of the wheels, if the car is brought to rest in 150m.

Solution

$\boxed{\begin{array}{l} V = r\omega \\ a = r\alpha \end{array}}$

$V = 65\ km/hr = 18.06\ m/s$

$r = 30cm = 0.3m$.

a) $\omega = \frac{V}{r} = \frac{18.06}{0.3} = \underline{\underline{60.2\ rad/s}}$

b) $u = 18.06\ m/s$

$v = 0$

$s = 150m$.

$v^2 - u^2 = 2as$

$0 - (18.06)^2 = 2 \times a \times 150$

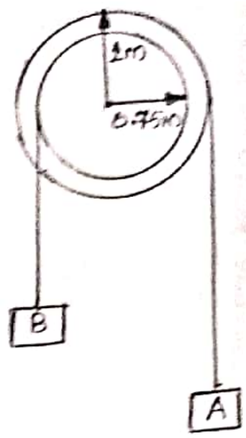
$a = -1.087\ m/s^2$

$a = r\alpha$.

$\therefore \alpha = \frac{a}{r} = \frac{-1.087}{0.3} = \underline{\underline{-3.623\ rad/s^2}}$

(7) The step pulley shown in figure starts from rest and accelerates at 2 rad/s^2. How much time is required for block A to move 20m ? Find also the velocity of A and B at that time.

Solution



when block A moves by 20m, θ is given by,

$S = r\theta$

$20 = 1 \times \theta$

$\therefore \theta = 20$ radians.

$\alpha = 2$ rad/s^2 ; $\omega_0 = 0$

$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$

$20 = 0 + \frac{1}{2} \times 2 \times t^2$

$t = \underline{\underline{4.472 \text{ s}}}$

Velocity of pulley at this time

$\omega = \omega_0 + \alpha t$

$= 0 + 2 \times 4.472 = \underline{\underline{8.944}}$ rad/s

Velocity of A ; $V_A = r\omega = 1 \times 8.944 = \underline{\underline{8.944}}$ m/s

Velocity of B ; $V_B = r\omega = 0.75 \times 8.944 = \underline{\underline{6.708 \text{ m/s}}}$

(8) A wheel of 1.2 m diameter starts from the rest and is accelerated at the rate of 0.8 rad/s^2. Find the linear velocity of a point on its periphery after 5 sec.

Solution

Radius, $r = \frac{1.2}{2} = 0.6$m

$\omega_0 = 0$

$\alpha = 0.8$ rad/s^2

$\omega = \omega_0 + \alpha t$

At $t = 5$ sec $\Rightarrow \omega = 0 + 0.8 \times 5 = 4$ rad/s.

$\therefore$ Linear velocity of the point on the periphery of wheel, $V = r\omega$

$= 0.6 \times 4$

$= \underline{2.4}$ m/s

⑨ A pulley 2m in diameter is connected to a shaft which makes 240 rpm. Find the linear velocity of a particle on the periphery of the pulley.

Solution

$r = \frac{2}{2} = \underline{\underline{1m}}$

$\omega = 240 \times \frac{2\pi}{60} = \underline{\underline{25.1\ rad/s}}$

Linear velocity, $V = r\omega = 1 \times 25.1 = \underline{\underline{25.1\ m/s}}$

⑩ A car travelling with a constant speed of 36 km/hr enters a curved portion of a road of radius 200m. Calculate the acceleration of the car in normal and tangential direction.

Solution

Since car is travelling with constant speed, $dv = 0$

$\therefore$ Tangential acceleration, $a_t = \frac{dv}{dt} = 0$ $V = 36\ km/h = \underline{\underline{10m/s}}$

Normal acceleration, $a_n = \frac{V^2}{r} = \frac{(10)^2}{200} = \underline{\underline{0.5\ m/s^2}}$

$$\boxed{\begin{aligned} a_t &= \frac{dv}{dt} = r\alpha \\ a_n &= V \cdot \omega = \frac{V^2}{r} \\ &= \omega^2 r \\ a &= \sqrt{a_t^2 + a_n^2} \\ \alpha &= \tan^{-1}\left(\frac{a_t}{a_n}\right) \end{aligned}}$$

(11) Starting from rest a particle moves along circular path of radius r, so that the distance travelled is given by the relation $s = kt^2$, where k is a constant. Find the tangential and normal component of the acceleration of a particle.

Solution

$\omega_0 = 0$

$s = kt^2$

$$V = \frac{ds}{dt} = 2kt$$

$$a_t = \frac{dv}{dt} = \underline{\underline{2k}}$$

$$a_n = \frac{v^2}{r} = \frac{(2kt)^2}{r} = \underline{\underline{\frac{4k^2t^2}{r}}}.$$

(12) A motorist starts from rest on a curve of radius 100m and accelerates at a uniform rate of $1 \, m/s^2$. Determine the distance travelled when the total acceleration reaches $2 \, m/s^2$.

Solution

$r = 100m$; $a_t = 1 m/s^2$

$a = 2 m/s^2$

$$a = \sqrt{a_t^2 + a_n^2}$$

$$2 = \sqrt{1^2 + a_n^2} \Rightarrow a_n = \sqrt{3} \, m/s^2$$

$$a_n = \frac{v^2}{r} \Rightarrow \sqrt{3} = \frac{v^2}{100} \Rightarrow V = \underline{\underline{13.16 m/s}}$$

$$v^2 - u^2 = 2a_t s \, ; \quad u = 0$$

$$(13.16)^2 - 0 = 2 \times 1 \times s$$

$$\therefore s = \underline{\underline{86.59m}}.$$

(13) A car enters a curve of $r = 450\,m$ at a speed of 19 km/hr. If the car increases its speed at a rate of $4.5\,m/s^2$, what will be its total acceleration when it has travelled 450 m along the curve?

Solution

$r = 450\,m$

$u = 19\,km/hr = 5.3\,m/s$

$a_t = 4.5\,m/s^2$

$s = 450\,m$

$v^2 - u^2 = 2a_t s$

$v^2 - (5.3)^2 = 2 \times 4.5 \times 450$

$V = 63.86\,m/s$

$$a_n = \frac{v^2}{r} = \frac{(63.86)^2}{450} = 9.06\,m/s^2$$

Total acceleration, $a = \sqrt{a_t^2 + a_n^2} = \sqrt{(4.5)^2 + (9.06)^2} = 10.11\,m/s^2$

(15) A car is travelling on a curved road of radius 1000 m at a speed of 180 km/hr. The brakes are suddenly applied causing the car to slow down at a uniform rate. After 10 sec, speed is reduced to 100 km/hr. Determine the deceleration immediately after the brakes are applied.

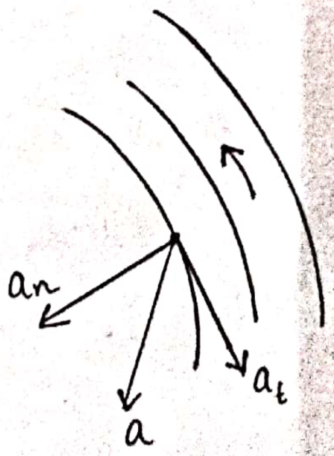
Solution

$r = 1000\,m$

$u = 180\,km/hr = 50\,m/s$

$V = 100\,km/hr = 27.7\,m/s$

$t = 10\,sec.$



$$V = u + a_t t \Rightarrow a_t = \frac{V-u}{t}$$

$$= \frac{27.7 - 50}{10} = -2.23\,m/s^2$$

$$a_n = \frac{V^2}{r} = \frac{(50)^2}{1000} = 2.5\,m/s^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-2.23)^2 + (2.5)^2} = 3.35\,m/s^2$$

(15) A horizontal bar of 1.5m length and of small c/s rotates about a vertical axis through one end. It accelerates uniformly from 1200 rpm to 1500 rpm in 5 seconds. What is the linear velocity at the beginning and at the end of 5 sec. What are the normal and tangential component of acceleration of the midpoint of the bar after 4 seconds?

Solution

$r = 1.5m$

$\omega_0 = 1200 \text{ rpm} = \frac{1200 \times 2\pi}{60} = \underline{40\pi \text{ rad/s}}$

$\omega = 1500 \text{ rpm} = \frac{1500 \times 2\pi}{60} = \underline{50\pi \text{ rad/s}}$

$t = 5 \text{ sec}$

Linear velocity at the beginning, $u = r\omega_0 = 1.5 \times 40\pi$

$= 60\pi = \underline{188.5 \text{ m/s}}$

Linear velocity at the end of 5 sec, $V = r\omega = 1.5 \times 50\pi$

$= 75\pi = \underline{235.62 \text{ m/s}}$

Tangential acceleration, $a_t = r\alpha$.

$\alpha = \frac{\omega - \omega_0}{t} = \frac{50\pi - 40\pi}{5} = \underline{2\pi \text{ rad/s}^2}$

a_t at the midpoint of the bar (ie $r_m = \frac{1.5}{2}$)

$a_t = r_m \alpha = \frac{1.5}{2} \times 2\pi = \underline{4.71 \text{ m/s}^2}$

Normal acceleration; $a_n = \omega^2 r$.

ω at the end of 4 sec $\Rightarrow \omega = \omega_0 + \alpha t$

$= 40\pi + 2\pi \times 4$

$= \underline{48\pi \text{ rad/s}}$

$\therefore a_n$ at the midpoint of the bar after 4 sec;

$a_n = r_m \omega^2 = \frac{1.5}{2} \times (48\pi)^2 = \underline{17054.676 \text{ m/s}^2}$

(16) A flywheel starts to rotate with uniform angular acceleration, reaches 2400 rpm in 40 sec. Determine the angular acceleration and no. of revolutions made by the flywheel after 2 sec. Also determine the linear velocity and total acceleration of a point on the rim of the flywheel after 2s. Diameter of the flywheel is 2m.

Solution

$\omega_0 = 0$; $t = 40$ sec

$\omega = 2400 \text{rpm} = \frac{2400 \times 2\pi}{60} = 80\pi$ rad/s

$\alpha = \frac{\omega - \omega_0}{t} = \frac{80\pi - 0}{40} = 2\pi$ rad/s^2

angular velocity after $t = 2$ sec; $\Rightarrow \omega = \omega_0 + \alpha t$

$= 0 + 2\pi \times 2$

$= 4\pi$ rad/s

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$.

$= 0 + \frac{1}{2} \times 2\pi \times (2)^2 = 4\pi$ rad.

No. of revolutions $= \frac{4\pi}{2\pi} = 2$

$r = \frac{2}{2} = 1$m.

Linear velocity, $V = r\omega = 1 \times 4\pi$

$= 12.56$ m/s

$a_t = r\alpha = 1 \times 2\pi = 2\pi = 6.283$ m/s^2

$a_n = r\omega^2 = 1 \times (4\pi)^2 = 157.91$ m/s^2

Total acceleration, $a = \sqrt{a_t^2 + a_n^2}$

$= \sqrt{(6.283)^2 + (157.91)^2}$

$= 158.03$ m/s^2

(17) A flywheel 0.5 m in diameter accelerates uniformly from rest to 360 rpm in 12 s. Determine the velocity and acceleration of a point on the ~~rim~~ rim of flywheel 0.1 s after it has started from rest.

Solution

$r = \frac{0.5}{2} = 0.25 m$

$\omega_0 = 0$

$\omega = 360 \, rpm = \frac{360 \times 2\pi}{60} = 12\pi \, rad/s$

$t = 12 s$

$\alpha = \frac{\omega - \omega_0}{t} = \frac{12\pi - 0}{12} = \pi \, rad/s^2$

angular velocity at $t = 0.1$ sec;

$\omega = \omega_0 + \alpha t = 0 + \pi \times 0.1 = 0.1\pi$ rad/s.

$\therefore$ linear velocity at $t = 0.1 s \Rightarrow V = r\omega$

$= 0.25 \times 0.1\pi$

$= 0.0785$ m/s

$a_t = r\alpha = 0.25 \times \pi = 0.785 \, m/s^2$

$a_n = r\omega^2 = 0.25 \times (0.1\pi)^2 = 0.0246 \, m/s^2$

Total acceleration; $a = \sqrt{(a_t)^2 + (a_n)^2}$

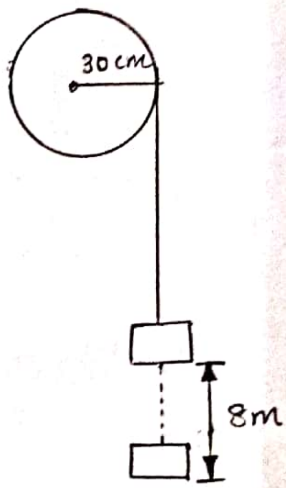
$= \sqrt{(0.785)^2 + (0.0246)^2}$

$= 0.785 \, m/s^2$

(18) A string is wrapped round a pulley of 60 cm diameter. One end is tied to the pulley and a weight is freely attached to the other end of the string. The weight travels a distance of 8 m within 4 s after attaching the weight. Find out
a) angular acceleration of pulley
b) final angular velocity of pulley
c) total distance travelled by the weight when pulley rotates at 300 rpm.

Solution.

$r = 60/2 = 30\,cm = 0.3\,m$



$\omega_0 = 0 \;;\; u = 0$

$S = ut + \frac{1}{2} \times a_t \times t^2$ (weight)

$8 = 0 + \frac{1}{2} \times a_t \times (4)^2$

$a_t = \underline{\underline{1\,m/s^2}}$

$a_t = r\alpha \Rightarrow \alpha = a_t/r = \frac{1}{0.3} = \underline{\underline{3.33}}\,rad/s^2$

a) $\therefore$ angular acceleration of pulley; $\alpha = \underline{\underline{3.33\,rad/s^2}}$.

b) Final angular velocity of pulley;

$\omega = \omega_0 + \alpha t = 0 + 3.33 \times 4 = \underline{\underline{13.32\,rad/s}}$.

c) Total distance travelled by the weight when pulley rotates at 300 rpm.

ie; $\omega = 300\,rpm = \frac{300 \times 2\pi}{60} = 10\pi\,rad/s$

$\omega^2 = \omega_0^2 + 2\alpha\theta$

$(10\pi)^2 = 0 + 2 \times 3.33 \times \theta$

$\therefore \theta = \underline{\underline{148.19}}\,rad.$

$\therefore S = r\theta = 0.3 \times 148.19 = \underline{\underline{44.46\,m}}.$

(19) A wheel of 120 cm diameter is mounted on a shaft and the shaft is supported b/w 2 bearings. The wheel is rotated from rest applying moment at the rim of the wheel and wheel starts rotating at 120 rpm within 12 min. Determine:

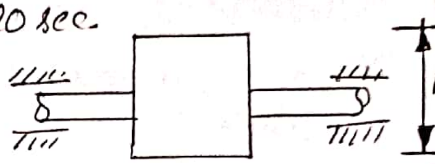
a) No of revolutions made by the wheel during this time.

b) Angular acceleration of the wheel

c) Peripheral velocity of the wheel in m/s at 120 rpm

Solution.

$r = \frac{120}{2} = 60cm = 0.6m$; $t = 12 min = 720 sec.$



$\omega_0 = 0$; $\omega = 120 rpm = \frac{120 \times 2\pi}{60}$

$= 4\pi\ rad/s$

b) angular acceleration, $\alpha = \frac{\omega - \omega_0}{t} = \frac{4\pi - 0}{720} = \frac{\pi}{180}\ rad/s^2$

$= 0.0174\ rad/s^2$

a) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$= 0 + \frac{1}{2} \times \frac{\pi}{180} \times (720)^2 = 1440\pi\ rad.$

No. of revolutions $= \frac{1440\pi}{2\pi} = 720$

c) Periphery velocity of the wheel, $V = r\omega$

$= 0.6 \times 4\pi$

$= 7.539\ m/s$

(20) An equation of motion of a body rotating along a circle of radius 10m is given by $s = 18t + 3t^2 - 2t^3$; where s is the distance covered from starting in meters and t is in seconds. Determine :-

a) the angular velocity and angular acceleration at start

b) the time when the body reaches maximum angular velocity.

c) the maximum angular velocity of the particle.

Solution

$S = 18t + 3t^2 - 2t^3$; $r = 10m$.

$V = \frac{ds}{dt} = 18 + 6t - 6t^2$

$a_t = \frac{dv}{dt} = 6 - 12t$

a) when $t = 0 \Rightarrow V = 18\,m/s$, & $a_t = 6\,m/s^2$.

but; $V = r\omega \Rightarrow \omega = V/r = \frac{18}{10} = \underline{\underline{1.8\,rad/s}}$.

$a_t = r\alpha \Rightarrow \alpha = a_t/r = \frac{6}{10} = \underline{\underline{0.6\,rad/s^2}}$.

b) when, $V = max$; $\frac{dv}{dt} = 0$

ie; $6 - 12t = 0 \Rightarrow \underline{\underline{t = 0.5\,sec}}$.

c) $V_{max} = 18 + 6 \times 0.5 - 6 \times (0.5)^2 = \underline{\underline{19.5\,m/s}}$

$\therefore \omega_{max} = \frac{V_{max}}{r} = \frac{19.5}{10} = \underline{\underline{1.95\,rad/s}}$.